Higher Physics

Our Dynamic Universe

Summary Notes

- Vectors
- Motion - Equations of Motion
- Motion - Graphs of Motion
- Forces, Energy and Power
- Collisions, Explosions and Impulse
- Gravitation
- Special Relativity
- The Expanding Universe

Vectors

Scalar quantities only have a magnitude (size).

Distance and speed are scalar quantities; as are time, frequency, energy and mass.

$$speed = \frac{distance}{time} \qquad v = \frac{d}{t}$$

Vector quantities have both magnitude (size) and direction.

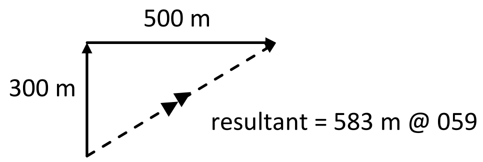
Velocity and displacement are vector quantities; as are acceleration and force.

$$velocity = \frac{displacement}{time} \qquad v = \frac{s}{t}$$

When adding vector quantities they must be added "nose-to-tail". This can be done by scale diagram or mathematics.

Right angled vectors:

e.g. A displacement of 300 m North then 500 m East



magnitude: Pythogoras ($a^2 = b^2 + c^2$)

$a^2 = 300^2 + 500^2$

$a = 583$ m

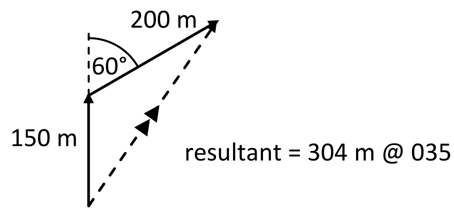
direction: trigonometry ($\tan\theta = \frac{opposite}{adjacent}$)

$\tan\theta = \frac{500}{300}$

$\theta = 59°$

Non-right angled vectors:

e.g. A displacement of 150 m North then 200 m bearing 060



magnitude: cosine rule ($a^2 = b^2 + c^2 - 2bc\cos A$)

$a^2 = 150^2 + 200^2 - (2 \times 150 \times 200 \times \cos 120)$

$a = 304$ m

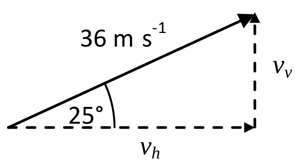
direction: sine rule ($\frac{a}{\sin A} = \frac{b}{\sin B}$)

$\frac{304}{\sin 120} = \frac{200}{\sin B}$

$B = 35°$

It is also often useful to separate a vector into **components**.

e.g. A velocity of 36 m s^{-1} at 25 to the horizontal



horizontal component $= 36\cos 25°$

$v_h = 32 \cdot 6$ m s^{-1}

vertical component $= 36\sin 25°$

$v_v = 15 \cdot 2$ m s^{-1}

Motion - Equations of Motion

Acceleration is the rate of change of velocity and is measured in metres per second per second (m s^{-2}).

When objects accelerate at a constant rate the **equations of motion** can be used to solve problems about their motion.

$$v = u + at$$

$$s = ut + \tfrac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

$$s = \frac{(u+v)}{2}t$$

s = displacement
u = initial velocity
v = final velocity
a = acceleration
t = time

Example 1:

A car is travelling at 15 m s^{-1}. After 5·0 s its velocity is 4·0 m s^{-1}. Calculate the acceleration of the car.

$s =$
$u = 15$ m s^{-1}
$v = 4\cdot0$ m s^{-1}
$a = ?$
$t = 5\cdot0$ s

$$v = u + at$$
$$4\cdot0 = 15 + (a \times 5\cdot0)$$
$$a = -2\cdot2 \text{ m s}^{-2}$$

i.e. a deceleration of 2·2 m s^{-2}

Example 2:

A train accelerates from rest at 0·40 m s^{-2} for 25 s. Calculate the displacement of the train during this time.

$s = ?$
$u = 0$ m s^{-1}
$v =$
$a = 0\cdot40$ m s^{-2}
$t = 25$ s

$$s = ut + \tfrac{1}{2}at^2$$
$$= (0 \times 25) + (\tfrac{1}{2} \times 0\cdot40 \times 25^2)$$
$$= 125 \text{ m}$$

Example 3:

A ball is thrown vertically upwards at a velocity of 12 m s^{-1}. Calculate the maximum height of the ball.

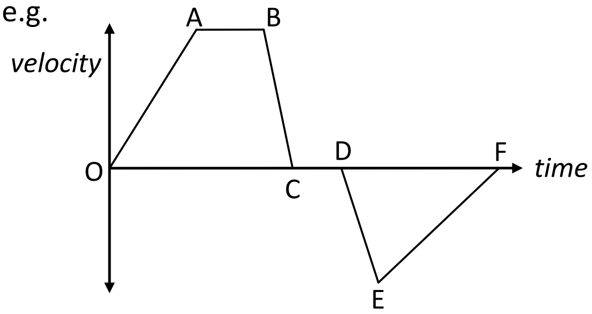
$s = ?$
$u = 12$ m s^{-1}
$v = 0$ m s^{-1}
$a = -9\cdot8$ m s^{-2}
$t =$

$$v^2 = u^2 + 2as$$
$$0^2 = 12^2 + (2 \times -9\cdot8 \times s)$$
$$19\cdot6\, s = 144$$
$$s = 7\cdot35 \text{ m}$$

Motion - Graphs of Motion

A **velocity-time graph** shows how the velocity of a moving object varies with time.

e.g.



OA – constant acceleration

AB – constant velocity

BC – constant deceleration

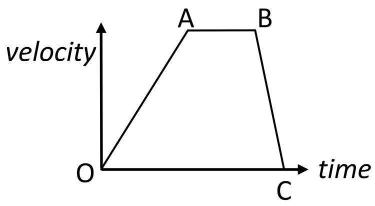
CD – at rest (zero velocity)

DE – constant acceleration in the opposite direction

EF – constant deceleration in the opposite direction

The acceleration of the object is equal to the gradient of the graph.

e.g.



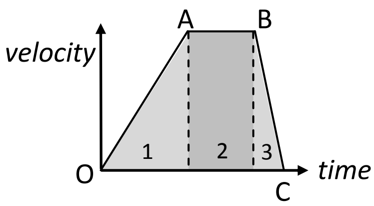
$$acceleration = gradient\ of\ graph$$

$$= \frac{change\ in\ velocity}{time}$$

Note: negative gradients represent negative accelerations

The displacement of the object is equal to the area under the graph.

e.g.



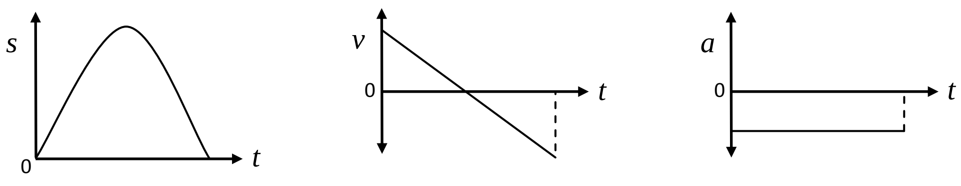
$$displacement = area\ under\ graph$$

$$= area\ 1 + area\ 2 + area\ 3$$

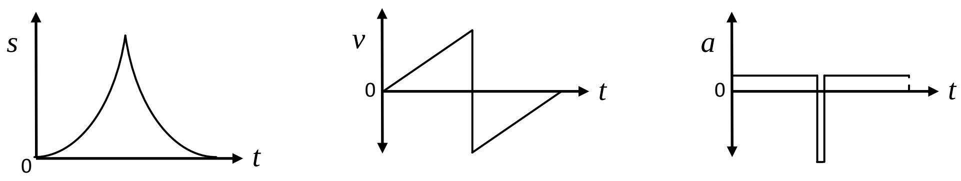
Note: areas below the time axis represent negative displacements (displacements in the opposite direction)

Displacement-time and acceleration-time graphs can also be drawn to represent the motion of objects.

e.g. ball thrown up (up is positive)



e.g. a ball bouncing (down is positive)



Forces, energy and power

The relationship between forces and motion are described by **Newton's Laws of Motion**.

Newton I: When the forces on an object are balanced, the object remains at rest, or moves at a constant speed in a straight line.

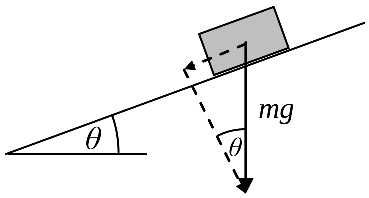
Newton II: When an unbalanced force is applied to an object the acceleration of the object is directly proportional to the size of the unbalanced force and inversely proportional to its mass.

$$F = ma$$

Newton III: When object A exerts a force on object B, object B exerts an equal and opposite force back on object A. ("*For every action there is an equal and opposite reaction*")

Newton's Laws can be used to solve problems involving multiple forces acting on an object:

Weight is the downward force on an object due to gravity ($W=mg$). When an object is on a slope the weight of the object can be resolved into two components; one acting parallel to the slope; and the other acting at right angles to the slope.



component down slope = $mg \sin \theta$

component at right angles to slope = $mg \cos \theta$

Friction is the force that acts between surfaces when they rub together. Friction always acts in the opposite direction to the relative motion between the surfaces.

Air resistance is the force that acts against an object as it moves through the air.

Tension is the pulling force of a string, rope, cable or chain.

Reaction is the force of contact from a surface. The reaction force always acts at right angles to the surface.

Other forces that may be involved in situations include: **thrust**, **engine force**, **upthrust** (buoyancy force), and **lift**.

To solve problems multiple forces can be resolved into a **resultant force**, which is a single force that has the same effect as the other forces combined.

Example 1:

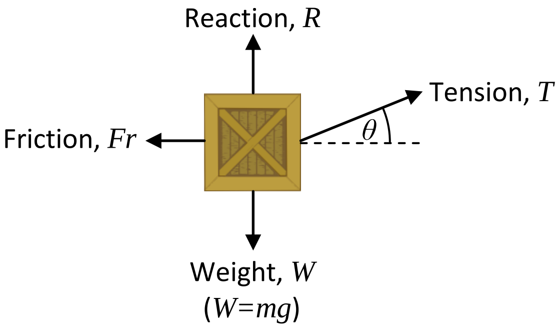
A 5·0 kg mass is suspended from a newton balance in a lift. As the lift is accelerating upwards the reading on the newton balance is 62 N. Calculate the acceleration of the lift.

$$\text{unbalanced force} = T - W$$
$$= 62 - (5\cdot0 \times 9\cdot8)$$
$$= 13 \text{ N}$$

$$F = ma$$
$$13 = 5\cdot0 \times a$$
$$a = 2\cdot6 \text{ m s}^{-2}$$

Example 2:

A 92 kg crate is pulled along a horizontal floor by a force of 82 N acting at 22° to the horizontal. There is a frictional force of 18 N acting on the crate. Calculate the acceleration of the crate.

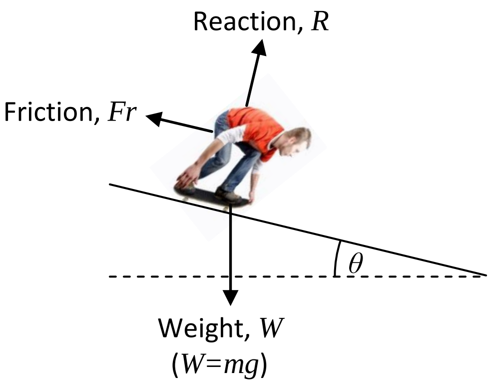


$$\text{unbalanced force} = T\cos\theta - Fr$$
$$= 82\cos 22 - 18$$
$$= 58\text{ N}$$

$$F = ma$$
$$58 = 92 \times a$$
$$a = 0 \cdot 63\text{ m s}^{-2}$$

Example 3:

A skateboarder accelerates down a 13° slope at 1·5 m s^{-2}. The mass of the skateboarder and board is 64 kg. Calculate the total frictional force acting on the skateboarder and board.

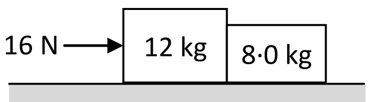


$$F = ma$$
$$= 64 \times 1 \cdot 5$$
$$= 96\text{ N}$$

$$\text{unbalanced force} = mg\sin\theta - Fr$$
$$96 = (64 \times 9 \cdot 8 \times \sin 13) - Fr$$
$$Fr = 45\text{ N}$$

Example 4:

Two boxes of mass 12 kg and 8·0 kg are placed next to each other on frictionless horizontal surface. A constant horizontal force of 16 N is applied to the 12 kg box towards the 8 kg box. Calculate the force of contact between the boxes.



For both boxes:

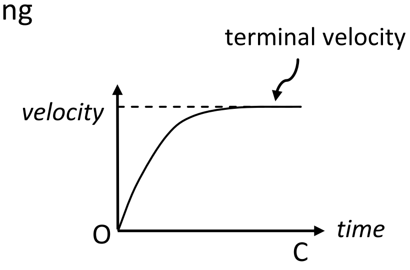
$$F = ma$$
$$16 = (12 + 8)a$$
$$a = 0 \cdot 8\text{ m s}^{-2}$$

For the 8·0 kg box:

$$F = ma$$
$$= 8 \times 0 \cdot 8$$
$$= 6 \cdot 4\text{ N}$$

As the velocity of an object increases the size of the frictional forces acting on it also increases. This effect means that as a falling object accelerates the unbalanced force on it decreases and so its acceleration also decreases. Eventually the object falls at a velocity where the forces on it become balanced. This is known as the **terminal velocity** of the object. The terminal velocity of an object is affected by factors such as its mass and surface area.

e.g. a skydiver falling



The **Principle of Conservation of Energy** states that *energy cannot be created or destroyed, but can only be converted from one form into another*. This principle can be used to solve situations involving energy changes:

gravitational potential energy	$E_p = mgh$
kinetic energy	$E_k = ½mv^2$
work done due to friction (heat)	$E_w = Fd$

Example:

A 850 kg car rolls 82 m down a slope. The slope is 8·4 m high. At the bottom of the slope the car is travelling at 3·2 m s^{-1}. Calculate the average frictional force acting on the car.

$$E_p = E_k + E_w$$
$$mgh = ½mv^2 + Fd$$
$$850 \times 9 \cdot 8 \times 8 \cdot 4 = (½ \times 850 \times 3 \cdot 2^2) + (F \times 82)$$
$$F = 800 \text{ N}$$

Power is the rate at which energy is transferred and is measured in watts (W).

$$P = \frac{E}{t}$$

Collisions, Explosions and Impulse

Momentum, p, is defined as the product of mass and velocity.

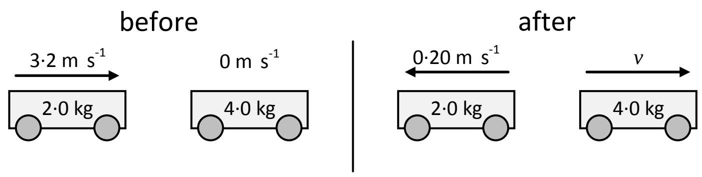
$$p = mv$$

The units of momentum are kg m s^{-1}. Momentum is a vector quantity.

During an interaction between objects (e.g. a collision or explosion) the total momentum before the interaction is equal to the total momentum after the interaction, providing there are no external forces acting on the objects. This principle is known as the **Law of Conservation of Momentum**.

Example 1:

A 2·0 kg trolley travelling at 3·2 m s^{-1} collides with a stationary trolley of mass 4·0 kg. After the collision the 2·0 kg trolley rebounds at a speed of 0·20 m s^{-1}. Calculate the velocity of the 4·0 kg trolley immediately after the collision.

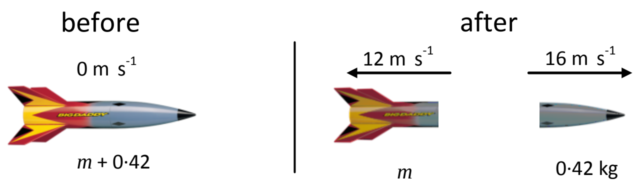


Total momentum before = total momentum after

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$
$$(2 \cdot 0 \times 3 \cdot 2) + (4 \cdot 0 \times 0) = (2 \cdot 0 \times -0 \cdot 2) + (4 \cdot 0 \times v)$$
$$6 \cdot 4 = -0 \cdot 4 + 4v$$
$$v = 1 \cdot 7 \text{ m s}^{-1}$$

Example 2:

A model rocket is launched vertically. At its maximum height, the rocket explodes into two parts. One part with a mass of 0·42 kg moves off at a velocity of 16 m s^{-1}. The other part moves off at velocity of 12 m s^{-1} in the opposite direction. Calculate the mass of the second part of the rocket.



Total momentum before = total momentum after

$$(m_1 + m_2) \times u = m_1v_1 + m_2v_2$$
$$(m + 0 \cdot 42) \times 0 = (m \times -12) + (0 \cdot 42 \times 16)$$
$$0 = -12m + 6 \cdot 72$$
$$m = 0 \cdot 56 \text{ kg}$$

In an **elastic** collision both momentum and kinetic energy are conserved.

In an **inelastic** collision momentum is conserved, but kinetic energy is not conserved.
(Note: any kinetic energy that is "lost" during an inelastic collision will be turned into heat)

To determine whether a collision is elastic or inelastic, calculate the total kinetic energy of the objects before the collision and the total kinetic energy of the objects after the collision.

$$E_k = ½mv^2$$

The concept of **impulse** is used to quantify the effect of an interaction between objects. Impulse is defined as the product of force and time.

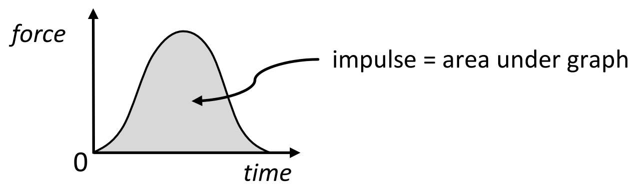
$$impulse = Ft$$

Impulse is also equal to the **change in momentum** of an object during an interaction.

$$Ft = mv - mu$$

Impulse has the units N s or kg m s^{-1}.

The force on an object during a real collision is unlikely to be constant, but impulse can also be determined from the area under a force-time graph.



During a collision between two objects the changes in momentum the object are equal in size and opposite and direction. Since the time of contact between the objects is the same for each object the forces acting on each object are also equal in size and opposite in direction (i.e. Newton's Third Law of Motion).

Gravitation

A **projectile** is an object that is in free fall, i.e. the only force acting on it is the force of gravity.

Projectile problems can be solved by separating the motion of the object into horizontal and vertical components:

horizontal motion : constant velocity using $s = vt$

vertical motion : constant downward acceleration using $v = u + at$
(Note: on Earth a = 9·8 m s^{-2})

The horizontal motion and vertical motion of a projectile are independent of each other.

A satellite is an object that is in free fall around a star or planet, but which has sufficient horizontal velocity that, as it accelerates toward the surface of the star or planet, the surface 'curves away' from it, so it never reaches the surface.

When an object with mass is placed in a gravitational field it experiences a force. The gravitational force acting on an object depends on the **mass,** m**,** of the object and the **gravitational field strength,** g**,** at that point. The gravitational force acting on an object is known as its **weight,** W.

$$W = mg$$

On the surface of the Earth, g = 9·8 N kg^{-1}, but this value varies according to altitude and the planet the object is on, as well as the specific geology of the location

Isaac Newton showed that gravitational is a universal force and that every object attracts every other object. Gravitation explains not only why objects accelerate towards the surface of the Earth, but also why moons stay in orbit around planets and why planets orbit stars.

The gravitational force, F, between any two objects is calculated using **Newton's Law of Universal Gravitation**.

$$F = \frac{Gm_1m_2}{r^2}$$

G = Universal Constant of Gravitation
m_1 = mass of first object
m_2 = mass of second object
r = distance between centres of masses

The value of the Universal Constant of Gravitation is 6·67 × 10^{-11} m^3 kg^{-1} s^{-2}.
(Note: the units N m^2 kg^{-2} are also acceptable)

This law is an 'inverse square law', as the size of the gravitational force varies with the inverse square of the distance between the centres of masses of the objects.

Gravitation is a mutual force between objects. The gravitational force of the first object on second object is equal, and in the opposite direction to, the gravitational force of the second object on the first object.

Special Relativity

In **classical relativity** observers in different frames of reference measure events with different results. For example a person walking on a train moving at 6 m s^{-1} relative to a platform might measure their velocity relative to the train to be 2 m s^{-1} in the direction of travel of the train, but an observer on the platform would observe them to be moving at 8 m s^{-1} relative to the platform.

Classical relativity is sufficient to explain most situations; however issues start to arise when we start to consider objects travelling at speeds approaching the speed of light. For example, imagine two spacecraft, each travelling at 2×10^8 m s^{-1}, towards each other. According to classical relativity, this would mean their relative velocity is 4×10^8 m s^{-1}. This is impossible, as nothing can travel faster than light. Also, the work of James Clark Maxwell had shown that the speed of light in a vacuum, whatever the speed of the observer, should be measured as 299 792 458 m s^{-1}.

In order to resolve these issues, Albert Einstein proposed his theory of **Special Relativity**.

The postulates of Special Relativity are:

1. When two observers are moving at constant speeds relative to one another, they will observe the same laws of physics.
2. The speed of light (in a vacuum) is the same for all observers.

A consequence of Special Relativity is that, in order to agree on the speed of light being the same, observers in different frames of reference will have to disagree about their measurements of time and distance.

Time dilation is the apparent increase in time of events on an object moving relative to an observer.

$$t' = t\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

t' = time measured by an observer, which the object is moving relative to
t = time measured in same frame of reference as the moving object
v = relative velocity
c = speed of light

Length contraction is the apparent decrease in length (in the direction of travel) of an object moving relative to an observer.

$$l' = l\sqrt{1-\frac{v^2}{c^2}}$$

l' = length measured by an observer, which the object is moving relative to
l = length measured in same frame of reference as the moving object
v = relative velocity
c = speed of light

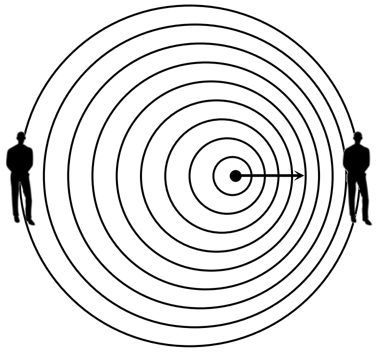
In practice, the effects of Special Relativity are only really noticeable when objects travel at speeds greater than 10% the speed of light (0·1c).

The Expanding Universe

The **Doppler Effect** causes shifts in wavelengths of sound and light as a source moves relative to an observer.

As a source moves towards an observer, the wavelength appears to be shorter and the frequency higher.

As a source moves away from an observer, the wavelength appears to be longer and the frequency lower.



The observed frequency of a wave from a moving source is given by the relationship:

$$f = f_0\left(\frac{v}{v \pm v_s}\right)$$

f = observed frequency
f_0 = source frequency
v = speed of wave
v_s = speed of source

Note: when moving towards use – ("take away")
when moving away use + ("add")

Light from objects moving away from us is shifted to longer (more red) wavelengths; it is **redshifted**. (Conversely, light from objects moving towards us is shifted to shorter (more blue) wavelengths; it is **blueshifted**.)

Redshift and blueshift are frequently observed in the light coming from stellar objects.

The shift in wavelength of absorption lines in the spectrum of light from stellar objects can be used to calculate their redshift, z.

$$z = \frac{\lambda_{observed} - \lambda_{rest}}{\lambda_{rest}}$$

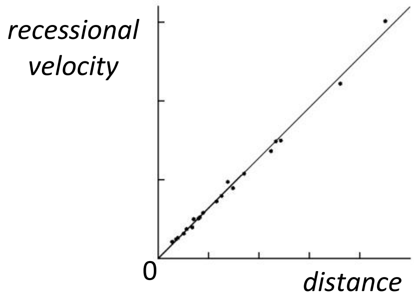
Note: negative values of redshift correspond to blueshift (i.e. approach velocity)

At low speeds, redshift is the ratio of the recessional velocity of the object, v, to the velocity of light, c:

$$z = \frac{v}{c}$$

Note: 'low speeds' means non-relativistic speeds, i.e. <0·1c

In the 1920s, the astronomer Edwin Hubble noticed that the redshift, and hence the recessional velocity, of distant galaxies is directly proportional to their distance from us.



This relationship is known as **Hubble's Law**.

$$v = H_0 d$$

The value of Hubble's constant, H_0, used in Higher Physics is $2{\cdot}3 \times 10^{-18}\ s^{-1}$.

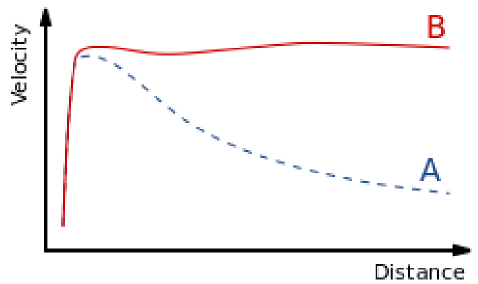
Hubble's measurements suggested that most galaxies are moving away from us and the further away they are, the faster they are moving. This suggests that the Universe is expanding.

If we assume that this expansion is at a steady rate, we can use Hubble's law to estimate the age of the Universe.

Since time, $t = \frac{d}{v}$ and we can rearrange Hubble's law as $\frac{d}{v} = \frac{1}{H_0}$ then, $t = \frac{1}{H_0}$

This suggests that the age of the Universe is approximately 14 billion years old.

The mass of a galaxy can be estimated by the orbital speed of the stars within it.



Observations (B) of the orbital speed of stars in galaxies show that stars in the outer parts of galaxies are moving much faster than is predicted (A) for the mass that is visible to us.

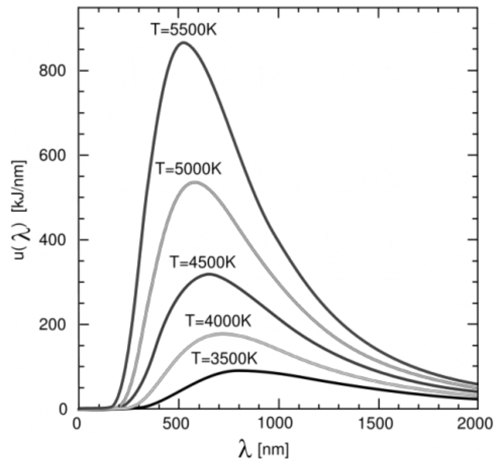
Therefore there must be a considerable amount of matter in galaxies that we cannot account for. We call this **dark matter**.

Recent observations of the redshift of distant galaxies show that not only is the Universe expanding, but that the rate of expansion is increasing. This suggests that there is some unknown force acting against gravitational attraction, which is pushing matter apart. As yet, astronomers and cosmologists have not been able to determine a source of energy capable of producing this force. For lack of a better term it is, for now, simply referred to as **dark energy**.

The temperature of stellar objects is related to the distribution of emitted radiation over a wide range of wavelengths. A graph of the emitted radiation against wavelength has a characteristic shape, known as a **black-body radiation curve**.

The **peak wavelength** of this distribution is shorter for hotter objects than for cooler objects.

At all wavelengths, the radiation emitted per unit surface area per unit time for hotter objects is greater than that for cooler objects.



Our current understanding is that the Universe started in an event known as the **Big Bang**. Initially the Universe was in a hot, dense state, but since that time has expanded and cooled.

Evidence for the Big Bang includes:

- Redshift of distant galaxies (Hubble's Law) – Hubble's measurements suggested that the Universe is expanding, so if we 'rewind time' there must have been a time when the Universe was at a single point.
- **Cosmic Microwave Background Radiation** (CMBR) – In 1964 Arno Penzias & Robert Wilson detected microwave radiation coming from all directions in space. It was realised that this was radiation left over from an early stage in the development of the Universe. Originally this radiation was at much shorter wavelengths, but the expansion of the Universe has caused the wavelength to increase over time and the peak wavelength is now in the microwave part of the spectrum.
- Olbers' Paradox – If the Universe is static; infinite; infinitely old; and contains an infinite number of stars, then a star should be visible in any line of sight and the night sky should therefore be white; but it is not.
- Proportions of hydrogen, helium and other elements – Although the Big Bang itself produced helium and lithium (Big Bang nucleosynthesis), all elements heavier than lithium can only be made by thermonuclear reactions in stars since the Big Bang (stellar nucleosynthesis). The relative proportions of the elements that we observe today correspond to what is predicted if the Universe started 14 billion years ago.